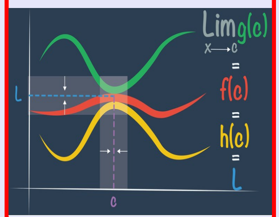
Calculus I

Lecture 18



Feb 19-8:47 AM

Class QZ 16

Use Substitution to evaluate

1) $\int x^{1/2}\sin(1+x^{3/2})\,dx$, $u=1+x^{3/2}$

$du=\frac{3}{2}x^{1/2}dx$

$\frac{2}{3}du=x^{1/2}dx$

$=\int \sin u\,\frac{2}{3}du=\frac{2}{3}\int \sin u\,du$

$=\frac{2}{3}\cdot -\cos u+C$

$=\boxed{-\frac{2}{3}\cos(1+x^{3/2})+C}$

2) $\int_0^{\pi/2}\cos x\cos(\sin x)\,dx$, $u=\sin x$

$du=\cos x\,dx$

$x=0 \quad u=\sin 0=0$

$x=\frac{\pi}{2} \quad u=\sin\frac{\pi}{2}=1$

$=\int_0^1 \cos u\,du$

$=\sin u\Big|_0^1=\sin 1-\cancel{\sin 0}^{\,0}$

$=\boxed{\sin 1}$

$\sin(\sin x)\Big|_0^{\pi/2}$

$=\sin(\sin\frac{\pi}{2})-\sin(\sin 0)$

$=\sin 1-\sin 0$

$=\boxed{\sin 1}$

$\cancel{\sin(\sin x)\Big|_0^1=\sin(\sin 1)-\sin(\sin 0)^{\,0}}$

$\cancel{=\sin(\sin 1)}$

Feb 4-7:41 AM

Open notes

Class QZ 15

1) Evaluate $\int_0^{\sqrt{\pi}} 2x \cos x^2 \, dx$

$u = x^2 \quad du = 2x\,dx$

$x = 0 \rightarrow u = 0$

$x = \sqrt{\pi} \rightarrow u = \pi$

$= \int_0^{\pi} \cos u \, du = \sin u \Big|_0^{\pi} = \sin \pi - \sin 0$

$= \boxed{0}$ ✓

2) Evaluate $\int (\tan x + 1)^3 \sec^2 x \, dx$

$u = \tan x + 1$

$du = \sec^2 x \, dx$

$= \int u^3 du = \frac{u^4}{4} + C$

$= \boxed{\frac{1}{4}(\tan x + 1)^4 + C}$

Feb 3-11:49 AM

Evaluate

1) $\int (1 - 2x)^9 dx$

$u = 1 - 2x \qquad \frac{du}{-2} = dx$

$du = -2\,dx$

$= \int u^9 \cdot \frac{du}{-2} = \frac{-1}{2} \int u^9 du = \frac{-1}{2} \cdot \frac{u^{10}}{10} + C$

$= \boxed{\frac{-1}{20}(1-2x)^{10} + C}$

2) $\int (2x+1)\sqrt{x^2+x} \, dx$

$u = x^2 + x$

$du = (2x+1)dx$

$= \int \sqrt{u}\, du = \int u^{1/2} du$

$= \frac{u^{3/2}}{3/2} + C = \frac{2}{3}(x^2+x)^{3/2} + C$

$= \boxed{\frac{2}{3}(x^2+x)\sqrt{x^2+x} + C}$

Feb 4-8:29 AM

3) $\int \frac{1(a+bx^2)}{\sqrt{3ax+bx^3}}\,dx$

$u = 3ax + bx^3$

$du = (3a + 3bx^2)dx$

$du = 3(a+bx^2)dx$

$\frac{du}{3} = (a+bx^2)dx$

$= \int \frac{1}{\sqrt{u}} \cdot \frac{du}{3} = \frac{1}{3}\int u^{-1/2}\,du$

$= \frac{1}{3} \cdot \frac{u^{1/2}}{1/2} + C$

$= \boxed{\frac{2}{3}\sqrt{3ax+bx^3} + C}$

4) $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

Hint: $u = 1 + \sqrt{x}$ $\rightarrow \sqrt{x} = u - 1$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$x = 0 \quad u = 1 + \sqrt{0} = 1$

$x = 1 \quad u = 1 + \sqrt{1} = 2$

$= \int_1^2 \frac{2\sqrt{x}\,du}{u^4}$

$= \int_1^2 \frac{2(u-1)\,du}{u^4}$

$= 2\int_1^2 \left[\frac{u}{u^4} - \frac{1}{u^4}\right] du = 2\int_1^2 (u^{-3} - u^{-4})\,du$

$= 2\left[\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3}\right]\Bigg|_1^2 = 2\left[\frac{1}{3u^3} - \frac{1}{2u^2}\right]\Bigg|_1^2$

$= 2\left[\left(\frac{1}{24} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right] = \boxed{}$

Feb 4-8:38 AM

$f(x)$ is continuous and $\int_0^4 f(x)\,dx = 10$,

find $\int_0^2 f(2x)\,dx$.

$u = 2x \qquad x = 0 \quad u = 2(0) = 0$

$du = 2\,dx \qquad x = 2 \quad u = 2(2) = 4$

$\frac{du}{2} = dx$

$= \int_0^4 f(u)\frac{du}{2}$

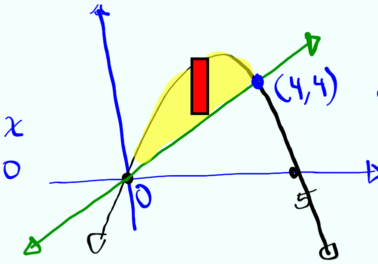
$= \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2} \cdot 10 = \boxed{5}$

Feb 4-8:58 AM

Find the area between $y=5x-x^2$ and $y=x$.

Parabola $\cap$

Line



$5x-x^2=x$

$4x-x^2=0$

$x=0$

$x=4$

$y=0$

$5x-x^2=0$

$x(5-x)=0$

$x=0 \quad x=5$

$$A=\int_0^4 [\text{Top}-\text{Bottom}]\,dx$$

$$=\int_0^4 [5x-x^2-x]\,dx$$

$$=\int_0^4 (4x-x^2)\,dx=\left(\frac{4x^2}{2}-\frac{x^3}{3}\right)\Big|_0^4$$

$$=\left(2x^2-\frac{x^3}{3}\right)\Big|_0^4=2\cdot 4^2-\frac{4^3}{3}-0$$

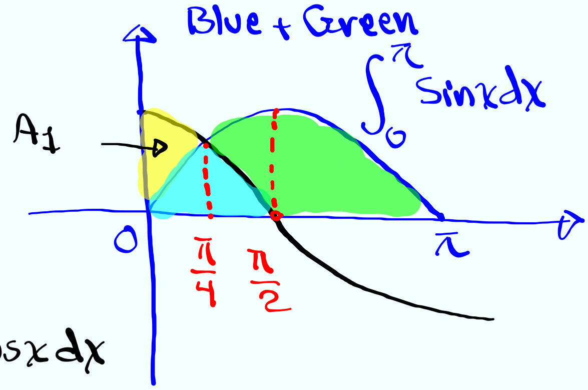
$$=32-\frac{64}{3}=\boxed{}$$

Feb 4-9:04 AM

Find the area between $y=\sin x$, $y=\cos x$ in QI.

Blue + Green

$$\int_0^{\pi} \sin x\,dx$$



$$A_1=\int_0^{\pi/4} (\cos x-\sin x)\,dx$$

$$A_2=\int_0^{\pi/4} \sin x\,dx+\int_{\pi/4}^{\pi/2} \cos x\,dx$$

$$A_3=\int_{\pi/4}^{\pi/2} (\sin x-\cos x)\,dx+\int_{\pi/2}^{\pi} \sin x\,dx$$

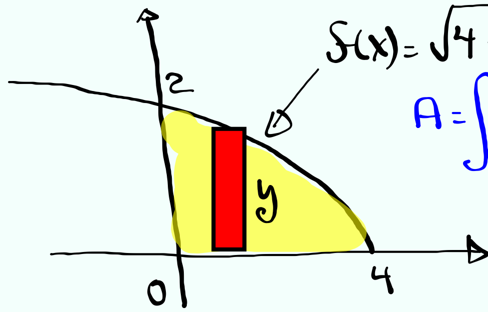
Feb 4-9:14 AM

Find the shaded area

$x=0 \rightarrow u=4-0=4$

$x=4 \rightarrow u=4-4=0$

$f(x)=\sqrt{4-x}$



$A=\int_0^4 \sqrt{4-x}\,dx$

$u=4-x$

$du=-dx$

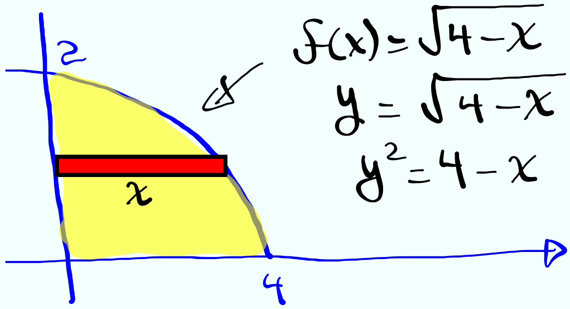
$-du=dx$

$A=\int_4^0 \sqrt{u}\cdot -du$

$= -\int_0^4 u^{1/2}\cdot -du = \dfrac{u^{3/2}}{3/2}\Big|_0^4 = \dfrac{2}{3}u\sqrt{u}\Big|_0^4$

$= \dfrac{2}{3}\cdot 4\sqrt{4} = \boxed{\dfrac{16}{3}}$

Feb 4-9:23 AM



$f(x)=\sqrt{4-x}$

$y=\sqrt{4-x}$

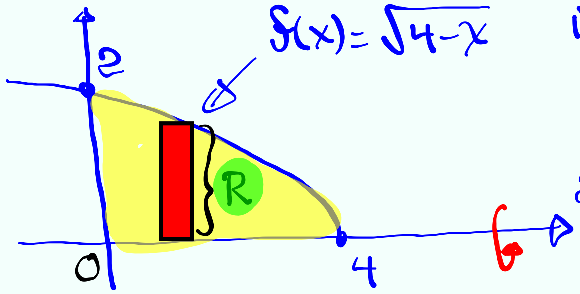
$y^2=4-x \rightarrow x=4-y^2$

$A=\int_0^2 (4-y^2)\,dy = \left(4y-\dfrac{y^3}{3}\right)\Big|_0^2 = 8-\dfrac{8}{3}$

$= \boxed{\dfrac{16}{3}}$

Feb 4-9:31 AM

Rotate the region shaded below about x-axis, find the volume.



1) Is region totally attached to A.O.R.?

2) Is Ref. Rect. $\perp$ A.O.R.?

Disk Method

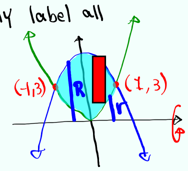
$$V=\int_0^4 \pi\left[\sqrt{4-x}\right]^2 dx$$

$$=\pi\int_0^4 (4-x)\,dx = \pi\left[4x-\frac{x^2}{2}\right]\Big|_0^4$$

$$=\pi[16-8]=\boxed{8\pi}$$

Feb 4-9:34 AM

Draw the region bounded by $y=4-x^2$ and $y=3x^2$. clearly label all intersection Points.



$3x^2=4-x^2$

$3x^2+x^2=4$

$4x^2=4$

$x^2=1 \quad x=\pm 1$

Rotate by x-axis, find volume.

Ref. Rect. $\perp$ A.O.R. ✓

Region is not totally attached to A.O.R

Washer Method

$$V=\int_a^b \pi\left[R^2-r^2\right]dx$$

$$V=\int_{-1}^{1}\pi\left[(4-x^2)^2-(3x^2)^2\right]dx$$

$$=\int_{-1}^{1}\pi\left[16-8x^2+x^4-9x^4\right]dx$$

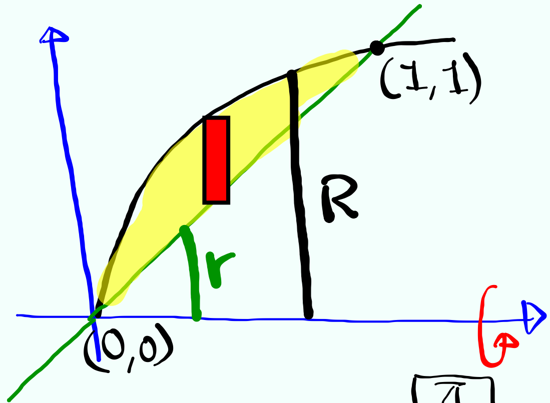
$$=\int_{-1}^{1}\pi\left[\overbrace{16-8x^2-8x^4}^{\text{even function}}\right]dx$$

$$=2\int_0^1 \pi\left(16-8x^2-8x^4\right)dx$$

$$=2\pi\left[16x-\frac{8x^3}{3}-\frac{8x^5}{5}\right]\Big|_0^1=2\pi\left[16-\frac{8}{3}-\frac{8}{5}-0\right]=\boxed{\frac{352\pi}{15}}$$

Feb 4-9:59 AM

Rotate the enclosed region by $y=\sqrt{x}$ and $y=x$ by the x-axis.



Washer Method

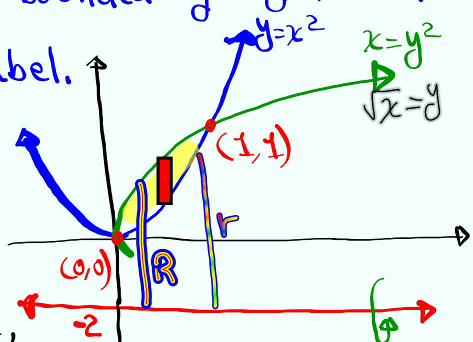
$$V=\int_a^b \pi\left[R^2-r^2\right]dx$$

$$=\int_0^1 \pi\left[(\sqrt{x})^2-(x)^2\right]dx$$

$$=\pi\int_0^1 (x-x^2)\,dx=\pi\left[\frac{x^2}{2}-\frac{x^3}{3}\right]\Bigg|_0^1=\pi\left[\frac{1}{2}-\frac{1}{3}\right]=\boxed{\frac{\pi}{6}}$$

Feb 4-10:12 AM

Draw the region bounded by $y=x^2$ and $x=y^2$. clearly label.



$R=y_{Green}+2=\sqrt{x}+2$

$r=y_{Blue}+2=x^2+2$

Rotate about $y=-2$, Set-up the integral for the volume.

Washer Method

$$V=\int_0^1 \pi\left[R^2-r^2\right]dx$$

$$=\int_0^1 \pi\left[(\sqrt{x}+2)^2-(x^2+2)^2\right]dx$$

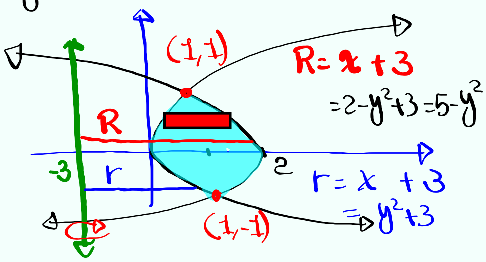
Feb 4-10:20 AM

Graph the region bounded by $x=y^2$ and $x=2-y^2$. Clearly label.

$y^2=2-y^2$

$2y^2=2$

$y=\pm 1$



Rotate about $x=-3$, set-up integrals for Volume.

washer method

$$V=\int_{-1}^{1}\pi\left[R^2-r^2\right]d=\pi\int_{-1}^{1}\underbrace{\left[(5-y^2)^2-(3+y^2)^2\right]}_{\text{even function}}dy$$

$$=2\pi\int_{0}^{1}\left[(5-y^2)^2-(3+y^2)^2\right]dy$$

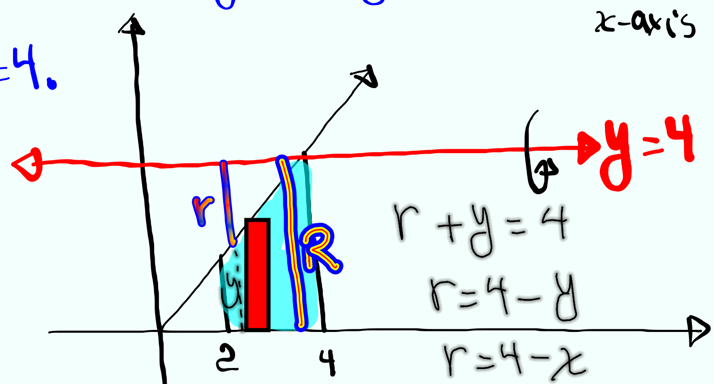
Feb 4-10:32 AM

Graph the enclosed region by $y=x$, $y=0$, x-axis, $x=2$, and $x=4$.

washer

$R=4$

$r=4-x$



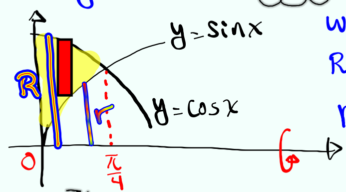
Rotate by $y=4$, Set-up integral for volume.

$$V=\int_{2}^{4}\pi\left[4^2-(4-x)^2\right]dx$$

Feb 4-10:46 AM

Consider the region enclosed by $y=\sin x$, $y=\cos x$, $0 \le x \le \frac{\pi}{4}$.

Rotate by x-axis, find the volume.



washer

$R = \cos x$

$r = \sin x$

$$V = \int_0^{\pi/4} \pi\left[\cos^2 x - \sin^2 x\right] dx$$

$$= \pi \int_0^{\pi/4} \cos 2x \, dx$$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$$= \pi \int_0^{\pi/2} \cos u \, \frac{du}{2}$$

$$= \frac{\pi}{2} \int_0^{\pi/2} \cos u \, du = \frac{\pi}{2} \sin u \Big|_0^{\pi/2}$$

$$= \frac{\pi}{2}\left[\sin\frac{\pi}{2} - \sin 0\right] = \boxed{\frac{\pi}{2}}$$

$\sin\frac{\pi}{2} \to 1$, $\sin 0 \to 0$

If $$f(x) = \int_{u(x)}^{v(x)} g(t)\,dt$$

then $$f'(x) = g(v(x))\cdot v'(x) - g(u(x))\cdot u'(x)$$

ex: $$f(x) = \int_1^{x^3} \sin t^2 \, dt$$

$$f'(x) = \sin(x^3)^2 \cdot 3x^2 - \sin(1^2)\cdot 0$$

$$= \boxed{3x^2 \sin x^6}$$

$$f(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^4+1}\,dt$$

$$f(1) = \int_1^{\sqrt{1}} \frac{t^2}{t^4+1}\,dt = \int_1^1 \text{whatever} = \boxed{0}$$

$$f'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1}\cdot\frac{1}{2\sqrt{x}} - \frac{1^2}{1^4+1}\cdot 0$$

$$= \boxed{\frac{x}{x^2+1}\cdot\frac{1}{2\sqrt{x}}}$$

Feb 4-11:11 AM

$$f(x) = \int_0^{x^4} \cos^2\sqrt{t}\,dt$$

1) $f(0) = \int_0^0 \text{whatever} = \boxed{0}$

2) $f'(x) = \cos^2\sqrt{x^4}\cdot 4x^3 - \cancel{\cos^2\sqrt{0}\cdot 0} \to 0$

$$= \boxed{4x^3\cos^2 x^2}$$

$$f(x) = \int_{x^2}^{x^3} \frac{t^2-1}{t^2+1}\,dt$$

1) $f(1) = \int_{1^2}^{1^3} \text{whatever} = 0$

2) find $f'(x) = \frac{(x^3)^2-1}{(x^3)^2+1}\cdot 3x^2 - \frac{(x^2)^2-1}{(x^2)^2+1}\cdot 2x$

$$= \frac{x^6-1}{x^6+1}\cdot 3x^2 - \frac{x^4-1}{x^4+1}\cdot 2x$$

Feb 4-11:14 AM

on what interval the curve of

$f(x)=\int_0^x \frac{t^2}{t^2+t+2}\,dt$ is $\boxed{\text{concave down}}$?

$(-4, 0)$

$f'(x)=\frac{x^2}{x^2+x+2}\cdot 1 - 0$

$f'(x)=\frac{x^2}{x^2+x+2}$

$f''(x)=\frac{2x(x^2+x+2)-x^2\cdot(2x+1)}{(x^2+x+2)^2}=\frac{\cancel{2x^3}+2x^2+4x\cancel{-2x^3}-x^2}{(x^2+x+2)^2}$

$=\frac{x^2+4x}{(\quad)^2}$ +

$x^2+4x=0$

$x(x+4)=0$

$x=0 \quad x=-4$

+	-4	−	0	+
CU		CD		CU

Feb 4-11:22 AM

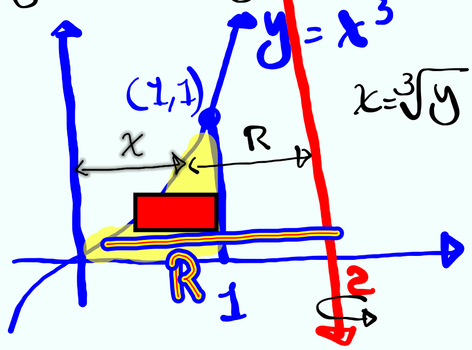
find the volume by rotating the enclosed region by $y=x^3$, $y=0$, and $x=1$ about $x=2$.

$r=1$

$x+R=2$

$R=2-x$

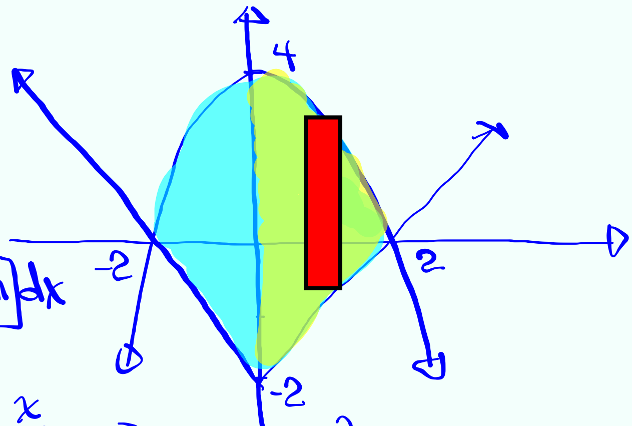
$=2-\sqrt[3]{y}$



$$V=\int_0^1 \pi\left[(2-\sqrt[3]{y})^2-1^2\right]dy$$

Feb 4-11:33 AM

Find the area enclosed by $y = 4 - x^2$ and $y = |x| - 2$.



$$A = 2\int_0^2 [\text{Top} - \text{Bottom}]\,dx$$

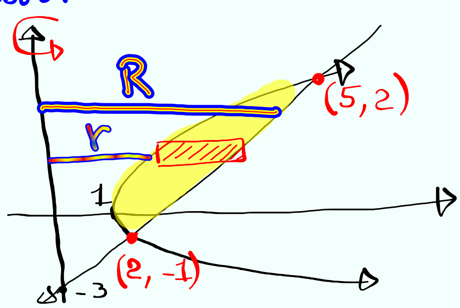
$$= 2\int_0^2 \Big[4 - x^2 - (\underbrace{|x|}_{x} - 2)\Big]dx = 2\int_0^2 [4 - x^2 - x + 2]\,dx$$

$$= 2\left[6x - \frac{x^3}{3} - \frac{x^2}{2}\right]\Big|_0^2 = 2\left[12 - \frac{8}{3} - 2\right] = \boxed{\frac{44}{3}}$$

Feb 4-11:43 AM

Rotate the region bounded by $x = 1 + y^2$ and $y = x - 3$ about the y-axis.

Find the volume.



$y = 1 + y^2 - 3$

$y^2 - y - 2 = 0$

$y = 2 \quad y = -1$

washer

$$V = \int_{-1}^{2} \pi [R^2 - r^2]\,dy$$

$R = x_{line} = y + 3$

$r = x_{curve} = 1 + y^2$

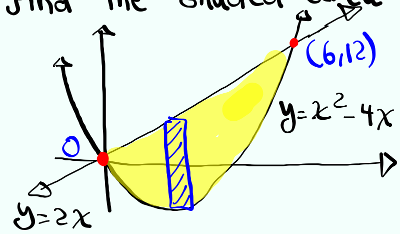
$$= \pi \int_{-1}^{2} \left[(y+3)^2 - (1+y^2)^2\right]dy$$

Feb 4-11:51 AM

Open notes

Class QZ 17

Find the shaded area below



$x^2-4x=2x$

$x^2-6x=0$

$x(x-6)=0$

$x=0 \quad x=6$

$$A=\int_0^6 [2x-(x^2-4x)]\,dx$$

$$=\int_0^6 (6x-x^2)\,dx$$

$$=\left(3x^2-\frac{x^3}{3}\right)\Big|_0^6$$

$$=3(6^2)-\frac{6^3}{3}=$$

$$=3\cdot 36-\frac{216}{3}$$

$$=108-72$$

$$=\boxed{36}$$

Feb 4-12:01 PM